Vector calculus in 3D

Below, $\vec{v} = [v_x, v_y, v_z]$ and $\vec{w} = [w_x, w_y, w_z]$ are 3D vectors and c is a scalar. Addition of vectors and multiplication ('scaling') of a vector by a scalar:

$$\vec{v} + \vec{w} = [v_x + w_x, v_y + w_y, v_z + w_z]$$
$$c\vec{v} = [cv_x, cv_y, cv_z].$$

Dot product is defined by

$$\vec{v} \cdot \vec{w} = v_x w_x + v_y w_y + v_z w_z.$$

Dot product of two vectors is zero if and only if they are perpendicular. Length of a vector $\vec{v}$, denoted by $|\vec{v}|$ can be computed as follows

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{\vec{v}^2}.$$

Dot product also provides a convenient way of computing (cosine of) the angle between two vectors because of the following formula.

$$\vec{v} \cdot \vec{w} = |v||w| \cos \angle(\vec{v}, \vec{w}).$$

Some properties of the dot product:

$$\vec{v} \cdot (\vec{w_1} + \vec{w_2}) = \vec{v} \cdot \vec{w_1} + \vec{v} \cdot \vec{w_2}$$
$$\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$$
$$(c\vec{v}) \cdot \vec{w} = \vec{v} \cdot (c\vec{w}) = c(\vec{v} \cdot \vec{w}).$$

Cross product of two vectors (which is also a 3D vector) can be computed from

$$\vec{v} \times \vec{w} = \left[\det\left[\begin{array}{cc} v_y & v_z \\ w_y & w_z \end{array}\right], \det\left[\begin{array}{cc} v_z & v_x \\ w_z & w_x \end{array}\right], \det\left[\begin{array}{cc} v_x & v_y \\ w_x & w_y \end{array}\right]\right].$$

The cross product $\vec{v} \times \vec{w}$ is perpendicular to both $\vec{w}$ and $\vec{v}$ and has magnitude of $|\vec{v}||\vec{w}| sin\angle(\vec{v}, \vec{w})$ (which happens to be the area of the parallelogram with edges running along $\vec{v}$ and $\vec{w}$). A few properties of the cross product:

$$\vec{v} \times \vec{w} = -\vec{w} \times \vec{v}$$
$$\vec{v} \times (\vec{w_1} + \vec{w_2}) = \vec{v} \times \vec{w_1} + \vec{v} \times \vec{w_2}$$
$$\vec{v} \times (c\vec{w}) = (c\vec{v}) \times \vec{w} = c(\vec{v} \times \vec{w})$$