1. You are clipping a triangle with respect to a rectangular viewport. Can the result have:

a) 3 edges?
b) 4 edges?
c) 5 edges?
d) 6 edges?
e) 7 edges?

For each case, either explain why or draw an example.

---

2. You are drawing two triangles, blue and red, using the z-buffer algorithm. The coordinates of their projected vertices and the corresponding depths are as follows:

blue: $(0, 0)$: depth $= 0.1$, $(5, 5)$: depth $= 0.6$, $(0, 5)$: depth $= 0.35$

red: $(5, 0)$: depth $= 0.2$, $(10, 0)$: depth $= 0.7$, $(0, 10)$: depth $= 0.7$

Knowing that it is within both projected triangles (I promise that, you don't have to verify), will the pixel $(3, 4)$ be colored a shade of blue or red? Assume the z-value is handled in the standard way (smaller means closer to the viewpoint).

---

3. When rendering a scene consisting of 77 spheres using a correctly written ray tracer, all of the spheres are at least partly visible from outside (you counted them!) and there are both specular and diffuse highlights on 20 of them. One of them can be seen even though it is completely in shadow.

Imagine meeting someone frustrated about his ray tracer, which persistently renders that scene as an entirely black image, even though he has spent two weeks chasing bugs and had not found a single new one for the last three days. Assuming that there is just one bug left, which of the following can it be (be sure to list *all* that are consistent with everything that has been said above):

1. The direction vector for the ray from the eye through the center of a pixel is wrong: it points in the opposite direction than it should

2. The diffuse component is not computed correctly.

3. The specular component is not computed correctly.

4. The ambient component is not computed correctly.

5. The shadow routine reports 'in shadow' all the time

6. The ray-sphere intersection routine reports 'no intersection' all the time

How would your answer change if the viewpoint were *inside* one of the spheres?

---

4. You have just got a viewer for 3D models, which is said to use just one shading model to render everything. Assume it does not show shadows.

What can this shading model be [be sure to list *all* shading models that could lead to this effect] if:
(a) there is a triangle over which the intensity is certainly not constant?
(b) you spotted a triangle that has a strong small highlight right in the middle?
(c) No matter what you view and where you put the light, all triangles appear to have constant intensity?
Shading models to choose from: Flat, Gouraud, Phong

---

5. Express the (planar) symmetry about the line passing through $(7, 8)$ and $(1, 2)$ as a 3×3 transformation matrix or as a product of 3×3 matrices (explain what they are).

---

6. An infinite cylinder is given by the equation

$$(x - a)^2 + (z - c)^2 = R^2.$$

Outline a procedure for computing the closest intersection point of a ray starting at $o = (x_o, y_o, z_o)$ and with a direction vector $d = (d_x, d_y, d_z)$ with that cylinder. Be sure to include and precisely state all the essential formulas. What is the formula for the normal vector at the intersection point?

---

7. Assume you shade the triangle using Gouraud shading (in OpenGL) and use true triangle normals at the vertices. Does the color of the triangle have to be constant? What if you were to use Phong Shading?

---

8. What is a triangle with vertices $(0, 0, 1)$, $(1, 2, 1)$, $(1, 0, 0)$ mapped to by the transformation with the following matrix in the homogenous coordinates

$$\begin{bmatrix} 1 & 1 & 3 & 1 \\ -1 & 0 & 0 & 2 \\ 5 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}?$$

How about the sphere centered at the origin and with radius 1 (write the equation of the resulting surface)?

---

9. The vertices of a triangle project to $(0, 0)$, $(1, 3)$ and $(3, 1)$. Let's say that the depth values (computed using the perspective transformation matrix) at these vertices are 1, 3 and 4 (respectively). What is the depth of the point on that triangle which projects to $(1, 1)$?

---

10. Can one obtain an 8-gon as a result of clipping a triangle with respect to a rectangle (in 2D)? How about an 10-gon resulting from clipping a triangle against a cube (in 3D)? Is it possible? Explain your answers.